Dissipation of a tide in a differentially rotating star

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ABSTRACT

The orbital period of the binary pulsar PSR J0045-7319, which is located in our neighbouring galaxy the Small Magellanic Cloud (SMC), appears to be decreasing on a timescale of half a million year. This timescale is more than two orders of magnitude smaller than what is expected from the standard theory of tidal dissipation. Kumar and Quataert (1997a) proposed that this rapid evolution can be understood provided that the neutron star's companion, a main sequence B-star, has set up significant differential rotation. They showed that the spin synchronization time for the B-star is similar to the orbit circularization time, whereas the time to synchronize the surface rotation is much shorter, and thus significant differential rotation in the star is indeed expected. However, their calculation did not include the various processes that can redistribute angular momentum in the star, possibly forcing it into solid body rotation; in that case the dissipation of the tide would not be enhanced. The goal of this paper is to include the redistribution of angular momentum in the B-star due to meridional circulation and shear stresses and to calculate the resulting rotation profile as a function of time. We find that although angular momentum redistribution is important, the B-star continues to have sufficient differential rotation so that tidal waves are entirely absorbed as they arrive at the surface. The mechanism proposed by Kumar and Quataert to speed up the orbital evolution of the SMC binary pulsar should therefore work as suggested.

Subject headings: stars: binary - stars: rotation

1. Introduction

An interesting pulsar binary system was discovered in a systematic search for pulsars in the Small Magellanic Cloud (SMC) by McConnell et al. (1991). This pulsar, PSR J0045-7319, has a spin period of 0.93 s and it is the only known pulsar in the SMC. Its companion is a main sequence B-star of mass $\sim 8.8\ M_\odot$ (Bell et al. 1995). The orbital period of this binary system is 51 days and its eccentricity is 0.808. According to Kaspi et al. (1996), the orbital period of this system is decreasing on a time scale of $\sim 5\times10^5$ years, whereas the standard theory for tidal dissipation predicts an evolution time of order 10^9 years (much of our understanding of dynamical tide is due to a series of papers by J.-P. Zahn, cf. Zahn 1975). The problem with the standard tidal theory, as applied to a rigidly rotating B-star of the SMC binary, is that only a small fraction of the tidal energy is dissipated in one orbital period. Kumar and Quataert (1997a, hereafter KQ) pointed out that if the entire tidal energy were to be dissipated in about one orbital period then the resulting evolution of the orbit would be consistent with the observation. They suggested that this can be accomplished provided that the rotation in the interior of the B-star deviates significantly from the rigid body rotation. This is in fact what is predicted by the work of Goldreich and Nicholson (1989) who showed that the angular momentum carried by tidal waves is deposited first near the surface of the star where the waves are dissipated. Thus, the surface can be brought into synchronous rotation on a time scale short compared to the time it takes for the star as a whole to become synchronous. The time scale for the latter process for the SMC pulsar binary system is shown to be comparable to the orbital circularization time (Lai 1996, Kumar and Quataert 1997a) and thus the star is not expected to be rotating synchronously; the observed apsidal motion of the star provides support for this picture (see Lai et al. 1995, Kaspi et al. 1996). However, if angular momentum is efficiently redistributed in stellar interiors, then a star can continue to rotate rigidly inspite of the tidal torque operating at the surface. The goal of this paper is to access the efficiency of angular momentum redistribution and to calculate the expected magnitude of differential rotation for the B-star of the SMC binary system. In §2, we describe our models and the physical mechanisms considered in this study. The main results are summarized in §3.

2. Angular velocity profile of a star due to tidal torque, meridional circulation and shear stresses

We consider the evolution of the rotation profile of a main sequence B-star of 9 $M_\odot$. We consider the star to be rotating uniformly until such a time so that the radius of the B-star, as a result of the normal stellar evolution, becomes $\sim 6\ R_\odot$. Fig. 1 shows the radius of the star and the size of its convective core as function of time.

The energy in the dynamical tide depends on the rotation rate of star. The tidal energy in a B-star, for parameters corresponding to the SMC binary system, and angular rotation speed perpendicular to the orbital plane of -1.6×10^{-5} rad s^{-1} (6×10^{-6} rad s^{-1}), is about 6×10^{40} erg (10^{40} erg) (see KQ); the minus sign refers to retrograde rotation. The average angular momentum luminosity associated with the tidal gravity wave for these two cases are 3×10^{38} gm cm^2 s^{-2} (10^{38} gm cm^2 s^{-2}) respectively.

The dominant process for tidal wave absorption in early type stars is the radiative damping which arises when photons diffuse from regions that are compressed to regions that are expanding. Since the photon mean free path increases rapidly with distance from the center of the star, most of the dissipation of the wave occurs close to the stellar surface. The fraction of wave momentum luminosity absorbed can be calculated using $f_{\rm damp} = \exp\left[\tau(r_u)\right]$ where

$$\tau(\omega,\ell,r) = [\ell(\ell+1)]^{\frac{3}{2}} \int_{r_l}^{r} K\, \frac{N N_t^2}{\omega^4} \left(\frac{N^2}{N^2-\omega^2}\right)^{\frac{1}{2}} \frac{{\rm d}r}{r^3}, \tag{1}$$

$\ell = 2$ for the quadrupole tide, $K = c/\rho\kappa$ is the thermal diffusivity, r_l is the lower turning point defined by $N(r_l)=\omega$, r_u is the upper turning point defined by $r_u = 6^{\frac{1}{2}} c_s/\omega$ ($r_l < r_u$), c_s is the sound speed, c is the speed of light, and N is the Brunt-Väisälä frequency which can be written as a sum of two parts, one of which is depends to the thermal gradient (∇) and the other on the gradient of the mean molecular weight (∇_μ) i.e.

$$N^2 = N_t^2 + N_\mu^2 = \frac{g\delta}{H_P}(\nabla_{\rm ad}-\nabla) + \frac{g\varphi}{H_P}\nabla_\mu \tag{2}$$

(see Unno et al. 1989, or Zahn et al. 1997 for details). The local luminosity of the wave is then given by

$$\mathcal{L}_J(\omega,\ell,r) = \mathcal{L}_J(\omega,\ell,r_l)\, \exp[-\tau(\omega,\ell,r)]. \tag{3}$$

If the star is rotating as a solid body, only the waves of frequency less than about 1.2×10^{-5} rad s^{-1} are completely absorbed as they reach the surface. For higher frequencies, the fraction of the wave flux absorbed decreases as $\sim \omega^{-5}$. However, differential rotation modifies that picture. Then, the frequency for the quadrupole tidal wave in the local rest frame of the fluid is $\omega(r) = \omega_t - 2\Omega(r)$, where $\Omega(r)$ is the angular speed of the star, $\omega_t \approx 2\Omega_p$ is the tidal frequency as seen in the inertial frame, and Ω_p is the orbital angular speed of the star at periastron. In a differentially rotating star, the local tidal wave frequency decreases outward and the wave dissipation rises rapidly. As waves deposit their angular momentum near the surface, the "synchronized front" starts at the surface and progresses inwards. However, one must also consider the role of the redistribution of angular momentum throughout the star as a result of meridional circulation and shear stress. These processes are calculated using the procedure described in Talon et al. (1997).

The evolution of the angular momentum in the model is governed by

$$\rho \frac{\mathrm{d}}{\mathrm{d}t}\left[r^2\Omega\right] = \frac{1}{5r^2}\frac{\partial}{\partial r}\left[\rho r^4 \Omega U\right] + \frac{1}{r^2}\frac{\partial}{\partial r}\left[\rho \nu_v r^4 \frac{\partial \Omega}{\partial r}\right] - \frac{3}{8\pi}\frac{1}{r^2}\frac{\partial}{\partial r}\mathcal{L}_J(r) \qquad (4)$$

where ρ is the density, ν_v is the vertical (turbulent) viscosity and U is the amplitude of the vertical circulation speed. The latter is calculated considering differential rotation in radius, as described in Zahn (1992) and in Matias et al. (1997), neglecting the role of the mean molecular weight gradients. The gradient of mean molecular weight would somewhat increase the timescale for the redistribution of momentum by the meridional circulation in the central part of the star. However, it would influence only marginally the results for the outer portion, which is the main object of this paper. The turbulent viscosity takes into account the weakening effect of the thermal diffusivity as first described by Townsend (1958). The details concerning that viscosity are not important here since the circulation, being an advective process, dominates the transport of momentum almost everywhere.

The local time scale that determines the redistribution of angular momentum due to meridional circulation is the Eddington-Sweet time scale. Fig. 2 shows this time scale as a function of r in the radiative part of a $9M_\odot$ main sequence star of age 19.7 Myrs. Since the Eddington-Sweet timescale is much shorter in the outer portion of the star, we expect the outer region to be rotating almost as a solid body whereas differential rotation can persist in the inner part[1].

Equation (4) is solved to determine the rotation profile as a function of time. For the model with mild retrograde solid body rotation, the upper turning point of the tidal wave is located at a radius of $0.85R_*$ and only about 1.0 % of its angular momentum luminosity is deposited during one orbital period. In this case, meridional circulation is efficient in redistributing the angular momentum in the interior of the star. In fact, as long as less than $\sim$ 30 % of the angular momentum luminosity of the tidal wave is deposited in the star, meridional circulation carries away almost all of this angular momentum and spreads it over much of the star, and thus, little differential rotation can be maintained. When wave damping is inefficient, the result of the rotation evolution calculation is dependent on the uncertain initial rotation profile of the star, and the time dependence of the angular momentum luminosity of the tide.

For these reasons we have carried out a restricted evolution calculation. We start with a state of differential rotation in the star so that a substantial fraction of the angular momentum luminosity of the tidal wave is deposited in the outer part of the star. We then calculate how the meridional circulation, and shear turbulence modify the rotation profile. In particular we wish to find an initial rotation profile leading to an increase of the differential rotation; a decrease in the differential rotation would decrease the rate of energy and angular momentum deposit and thus increase the time scale for the orbital evolution. To this end our calculation was started with the rotation profile shown in Fig. 3a (at 20 Myrs). In that case, the rate of angular momentum deposit was about 50 % of the luminosity. The initial rotation profile for prograde rotation was solid-body, as shown in Fig. 3b; in this case, about 80 % of the angular momentum luminosity of the wave is deposited near the upper turning point (at $\sim$ 0.96 R$_*$). The subsequent evolution of the rotation profile in each of these two cases is shown in Fig. 3; note that the difference between the rotation rate in the core and

[1] Actually, meridionnal circulation does not tend to produce solid body rotation but rather a state of mild differential rotation with $\Omega_{\mathrm{core}}/\Omega_{\mathrm{surface}} \sim 1.2$ for a 9 $M_\odot$ star (see Talon et al. 1997 for more details).

the surface is increasing with time until the surface is nearly synchronized. In Fig. 4, we show the wave damping as a function of r for every rotation profile shown in Fig. 3. Note that in all of these cases the tidal wave energy is completely dissipated as the wave approaches the stellar surface, and thus the angular momentum transported by the meridional circulation and shear induced turbulence does not modify the rate of change of the orbital energy for the SMC binary pulsar system PSR J0045-7319 estimated by Kumar and Quataert (1997). Thus, the time scale for the evolution of the orbital period calculated by these authors is unaffected by the redistribution of angular momentum in the star.

We have also calculated the effect of the differential rotation induced turbulence on the tidal wave damping. We find that it can be an important contributor to the wave damping at initial stages of evolution, when the surface of the star is rotating differentially, helping to enhance the fraction of tidal angular momentum luminosity deposited in the star.

We would like to point out one simplification made in our calculation. Even though the energy in the tidal waves is modified as the rotation profile of the star evolves we took it to be constant. However, the error made is not large since the tidal waves are mostly excited near the boundary of the convective core and radiative envelope where the rotation is nearly constant.

We note that the contribution to the wave damping from the region near the bottom of the radiative zone is negligible inspite of the fact that the mean molecular weight has a large gradient there leading to a sharp increase of the Brunt-Väisälä frequency. The reason is simply that the photon mean free path here is also very short and it takes a long time for photons to diffuse across a wavelength. Even if the dissipation in that region is about 5 times greater than just outside of it, the contribution to the total damping remains negligible.

3. Conclusion

We have calculated the evolution of the rotation profile of a 9 $M_\odot$ main sequence star which is subject to a tidal torque from its companion neutron star and in which angular momentum is redistributed by meridional circulation and shear stresses. We calculated models corresponding to two different rotation states: one model has a retrograde initial rotation of -1.6×10^{-5} rad s^{-1}, and the other has a prograde initial rotation of 6×10^{-6} rad s^{-1}. The magnitude of the tidal torque was taken to be 3×10^{38} gm cm^2 s^{-2} for the retrograde rotation and 10^{38} gm cm^2 s^{-2} for the prograde rotation. The location of the dissipation of the momemtum luminosity was calculated as a function of the rotation profile. The meridional circulation and shear stresses were calculated following the method described in Talon et al. (1997). We find that the fractional difference between the surface and the interior rotation rate in the star is a factor of a few. With this differential rotation, the frequency of tidal waves in the rest frame of the star is low enough that they are completely absorbed before they reach the upper turning point. For retrograde rotation of frequency about 6×10^{-6} rad s^{-1} or greater and a significant amount of differential rotation, the energy dissipated per orbit is about 10^{41} erg, which leads to orbit evolution time of $\sim 5 \times 10^5$ years, consistent with the observations.

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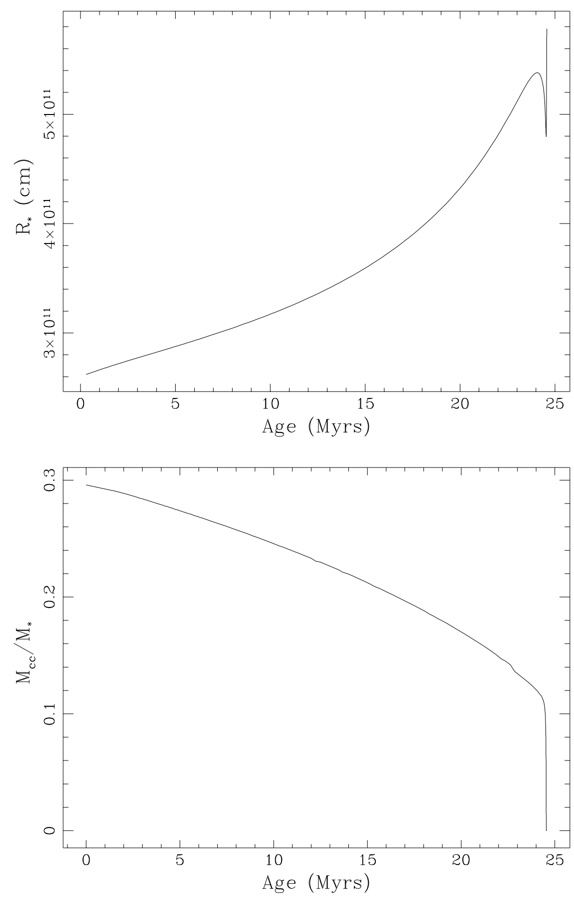

Fig. 1.— The radius of the B-star as a function of time is shown in the top panel, and the lower panel shows the size of the convective core (fractional mass) as the star evolves. The rapid change in the stellar radius and the mass of the convective core at about 24.5 Myrs is the because the star is evolving off the main sequence.

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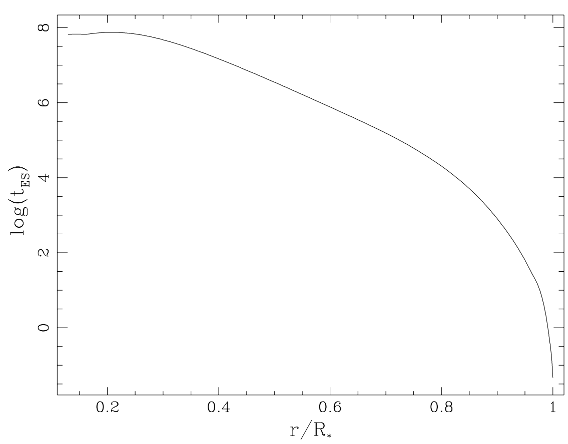

Fig. 2.— Local Local Eddington-Sweet timescale at radius r, $t_{ES}(r) = (GM_r^2/L_r)(4\pi G\rho(r)/3\Omega^2)$, characterizing the adjustment of the rotation profile through meridional circulation (in years) for a 9.0 $M_\odot$ main sequence star of age 19.7 Myrs.

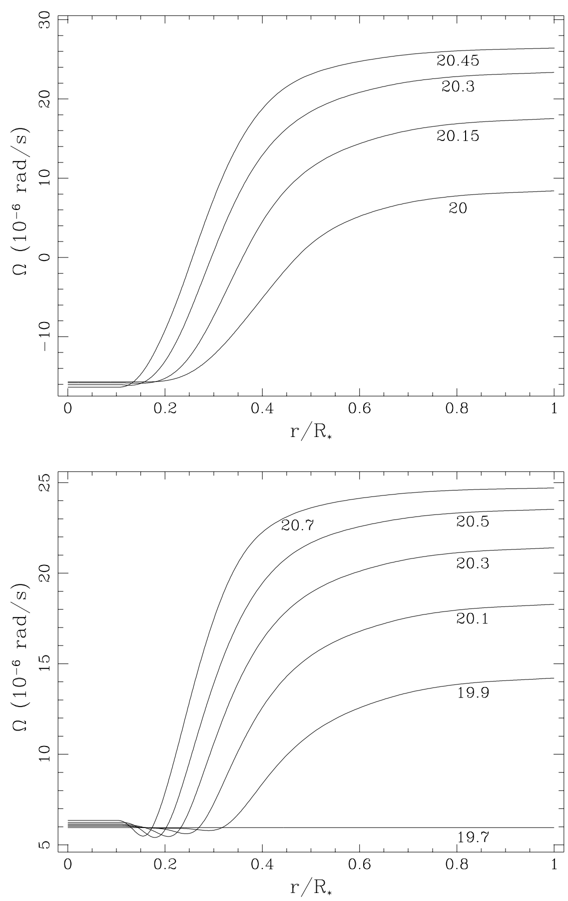

Fig. 3.— Rotation profile of the star at several times (in millions of years). The top panel shows the case where the initial rotation rate of the star was retrograde with angular speed of 1.6×10^{-5} rad s^{-1}, and the lower panel corresponds to the prograde rotation speed of 6×10^{-6} rad s^{-1}.

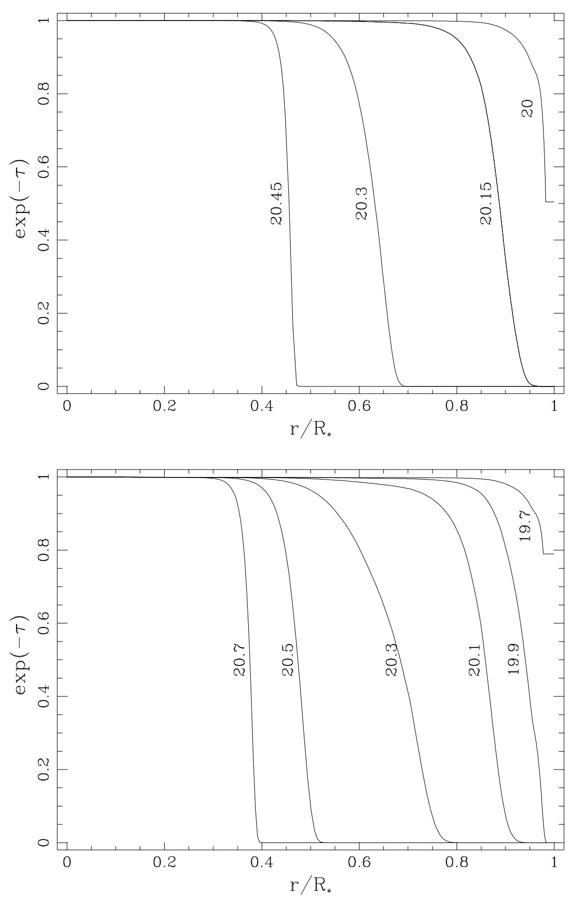

Fig. 4.— Relative magnitude of the momentum luminosity $\mathcal{L}(r)/\mathcal{L}(r_c) = \exp(-\tau)$ as a function of time for the two different cases of initial rotation of the B-star: retrograde rotation with angular speed 1.6×10^{-5} rad s^{-1}, and prograde rotation rate of 6×10^{-6} rad s^{-1}. Note that with increasing time, as the rotation profile evolves and the thinkness of the synchronously rotating shell near the surface increases, the wave dissipation occurs at smaller r.